

Are you ready for Calculus? Solutions

$$1. a. \quad \frac{x(x^2 - 9)}{(x-4)(x-3)} = \frac{x(x-3)(x+3)}{(x-4)(x-3)} = \frac{x(x+3)}{x-4}$$

$$1. b. \quad \frac{x^2 - 2x - 8}{x^3 + x^2 - 2x} = \frac{(x-4)(x+2)}{x(x^2 + x - 2)} = \frac{(x-4)(x+2)}{x(x-1)(x+2)} = \frac{x-4}{x(x-1)}$$

$$1. c. \quad \left[\frac{\frac{1}{x} - \frac{1}{5}}{\frac{1}{x^2} - \frac{1}{25}} \right] \frac{25x^2}{25x^2} = \frac{(25x - 5x^2)}{(25 - x^2)} = \frac{5x(5 - x)}{(5 - x)(5 + x)} = \frac{5x}{5 + x}$$

$$1. d. \quad \left[\frac{9 - \frac{1}{x^2}}{3 + \frac{1}{x}} \right] \frac{x^2}{x^2} = \frac{9x^2 - 1}{3x^2 + x} = \frac{(3x-1)(3x+1)}{x(3x+1)} = \frac{3x-1}{x}$$

$$2. a. \quad \left(\frac{2}{\sqrt{3} + \sqrt{2}} \right) \left(\frac{\sqrt{3} - \sqrt{2}}{\sqrt{3} - \sqrt{2}} \right) = \frac{2(\sqrt{3} - \sqrt{2})}{3 - 2} = 2(\sqrt{3} - \sqrt{2})$$

$$2. b. \quad \left(\frac{4}{1 - \sqrt{5}} \right) \left(\frac{1 + \sqrt{5}}{1 + \sqrt{5}} \right) = \frac{4(1 + \sqrt{5})}{1 + \sqrt{5} - \sqrt{5} - 5} = \frac{4(1 + \sqrt{5})}{-4} = -(1 + \sqrt{5}) = -1 - \sqrt{5}$$

$$\begin{aligned} 2. c. \quad & \left(\frac{1}{(1 + \sqrt{3}) - \sqrt{5}} \right) \left(\frac{(1 + \sqrt{3}) + \sqrt{5}}{(1 + \sqrt{3}) + \sqrt{5}} \right) = \frac{(1 + \sqrt{3}) + \sqrt{5}}{(1 + \sqrt{3})^2 - (\sqrt{5})^2} = \frac{(1 + \sqrt{3}) + \sqrt{5}}{1 + 2\sqrt{3} + 3 - 5} \\ & = \left(\frac{1 + \sqrt{3} + \sqrt{5}}{-1 + 2\sqrt{3}} \right) \left(\frac{-1 - 2\sqrt{3}}{-1 - 2\sqrt{3}} \right) = \frac{-1 - 2\sqrt{3} - 1\sqrt{3} - 6 - 1\sqrt{5} - 2\sqrt{15}}{1 - 12} \\ & = \frac{-1 - 2\sqrt{3} - 1\sqrt{3} - 6 - 1\sqrt{5} - 2\sqrt{15}}{1 - 12} = \frac{-7 - 3\sqrt{3} - \sqrt{5} - 2\sqrt{15}}{-11} \\ & = \frac{7 + 3\sqrt{3} + \sqrt{5} + 2\sqrt{15}}{11} \end{aligned}$$

$$3. a. \quad \frac{(2a^2)^3}{b} = \frac{8a^6}{b} \quad [= 8a^6b^{-1}]$$

$$3. b. \quad \sqrt{9ab^3} = (9ab^3)^{\frac{1}{2}} = 3a^{\frac{1}{2}}b^{\frac{3}{2}}$$

$$3. c. \quad \frac{\frac{2a}{\frac{b}{3}}}{a} = \frac{2a}{b} \cdot \frac{a}{3} = \frac{2a^2}{3b} \quad [= \frac{2}{3}a^2b^{-1}]$$

$$3. d. \quad \frac{a(b-1)}{b(b-1)} = \frac{a}{b} \quad [= ab^{-1}]$$

$$3. e. \quad \frac{b}{a\sqrt{a}} = \frac{b}{a^{\frac{3}{2}}} \quad [= a^{-\frac{3}{2}}b]$$

$$3. f. \quad \frac{a^{\frac{4}{3}}b^{\frac{3}{2}}}{ba^{\frac{1}{2}}} = \frac{a^{\frac{8}{6}}b^{\frac{3}{2}}}{ba^{\frac{3}{6}}} = a^{\frac{5}{6}}b^{\frac{1}{2}}$$

$$\begin{array}{lcl}
 4.a. & 5^{x+1} = 5^2 & 4.b. \quad 3^{-1} = 3^{2x+2} \\
 & x+1=2 & 2x+2 = -1 \\
 & x=1 & 2x = -3 \\
 & & x = -\frac{3}{2}
 \end{array}$$

$$4.c. \quad x = 2^3 = 8$$

$$\begin{array}{l}
 4.d. \quad \log_3 x^2 = \log_3 4^2 - \log_3 5^4 \\
 \log_3 x^2 = \log_3 \frac{4^2}{5^4} \\
 x^2 = \frac{4^2}{5^4} \\
 x = \pm \frac{4}{5^2} = \pm \frac{4}{25}
 \end{array}$$

$$5.a. \quad \log_2 \frac{5(x^2-1)}{x-1} = \log_2 \frac{5(x-1)(x+1)}{x-1} = \log_2 5(x+1)$$

$$\begin{aligned}
 5.b. \quad \log_4 9^2 - \log_2 3 &= \frac{\log_2 9^2}{\log_2 4} - \log_2 3 = \frac{\log_2 9^2}{2} - \log_2 3 = \log_2 (9^2)^{\frac{1}{2}} - \log_2 3 \\
 &= \log_2 9 - \log_2 3 = \log_2 \frac{9}{3} = \log_2 3
 \end{aligned}$$

$$5.c. \quad 3^{\log_3 5^2} = 25 \quad 6.a. \quad \frac{1}{2} \quad 6.b. \quad \log_{10} (10^{-x}) = -x$$

$$6.c. \quad \log_{10} (\sqrt{x})^2 + \log_{10} \left(x^{\frac{1}{3}} \right)^3 = \log_{10} x + \log_{10} x = 2\log_{10} x \quad [= \log_{10} x^2]$$

$$7.a. \quad \frac{x}{a} + \frac{y}{b} + \frac{z}{c} = 1$$

$$\frac{x}{a} = 1 - \frac{y}{b} - \frac{z}{c}$$

$$x = a \left[1 - \frac{y}{b} - \frac{z}{c} \right]$$

$$a = \frac{x}{\left[1 - \frac{y}{b} - \frac{z}{c} \right]} \left[\frac{bc}{bc} \right]$$

$$a = \frac{bcx}{bc - cy - bz}$$

$$7.b. \quad \frac{V}{2} = ab + bc + ca$$

$$\frac{V}{2} - bc = ab + ac$$

$$\frac{V}{2} - bc = a(b + c)$$

$$a = \left[\frac{\frac{V}{2} - bc}{b + c} \right] \left[\frac{2}{2} \right]$$

$$a = \frac{V - 2bc}{2(b + c)}$$

$$7.c. \quad A = 2\pi r^2 + 2\pi r h$$

$$0 = 2\pi r^2 + 2\pi r h - A$$

$$r = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$a = 2\pi \quad b = 2\pi h, \quad c = -A$$

$$r = \frac{-2\pi h \pm \sqrt{(2\pi h)^2 - 4(2\pi)(-A)}}{2(2\pi)}$$

$$= \frac{-2\pi h \pm \sqrt{4\pi^2 h^2 + 8A\pi}}{4\pi}$$

$$= \frac{-\pi h \pm \sqrt{\pi^2 h^2 + 2A\pi}}{2\pi}$$

only asks for positive r so

$$r = \frac{-\pi h + \sqrt{\pi^2 h^2 + 2A\pi}}{2\pi}$$

$$7.d. \quad A = P + nrP$$

$$A = P(1 + nr)$$

$$P = \frac{A}{1 + nr}$$

$$7.e. \quad 2x - y = xd + 2yd$$

$$2x - y = d(x + 2y)$$

$$d = \frac{2x - y}{x + 2y}$$

$$8.a. \quad y = (x^2 + 4x + 4) + 3 - 4$$

$$y = (x^2 + 4x + 4) + 3 - 4$$

$$y = (x + 2)^2 - 1$$

$$y + 1 = (x + 2)^2$$

$$y - (-1) = (x - (-2))^2$$

$$7.f. \quad \frac{2x}{4\pi} + \frac{1-x}{2} = 0$$

$$2\pi \left[\frac{x}{2\pi} + \frac{1-x}{2} = 0 \right]$$

$$x + \pi(1-x) = 0$$

$$x + \pi - x\pi = 0$$

$$x - x\pi = -\pi$$

$$x(1 - \pi) = -\pi$$

$$x = \frac{-\pi}{1 - \pi} = \frac{\pi}{\pi - 1}$$

$$8.b. \quad -2y = 3x^2 + 3x$$

$$-2y = 3(x^2 + x +)$$

$$-2y = 3\left(x^2 + x + \frac{1}{4}\right) - \frac{3}{4}$$

$$-2y + \frac{3}{4} = 3\left(x + \frac{1}{2}\right)^2$$

$$-2\left(y - \frac{3}{8}\right) = 3\left(x + \frac{1}{2}\right)^2$$

$$y - \frac{3}{8} = -\frac{3}{2}\left(x - \left(-\frac{1}{2}\right)\right)^2$$

$$8.c. \quad x + 9 = 9 \left(y^2 - \frac{2}{3}y \right)$$

$$x + 9 = 9 \left(y^2 - \frac{2}{3}y + \frac{1}{9} \right) - 1$$

$$x + 10 = 9 \left(y^2 - \frac{2}{3}y + \frac{1}{9} \right)$$

$$x - (-10) = 9 \left(y - \frac{1}{3} \right)^2$$

$$9.a. \quad x^6 - 16x^4 = x^4 (x^2 - 16) = x^4 (x + 4)(x - 4)$$

$$9.b. \quad 4x^3 - 8x^2 - 25x + 50 = 4x^2(x - 2) - 25(x - 2) = (x - 2)(4x^2 - 25) = (x - 2)(2x + 5)(2x - 5)$$

$$9.c. \quad 8x^3 + 27 = (2x + 3)(4x^2 - 6x + 9)$$

$$9.d. \quad x^4 - 1 = (x^2 + 1)(x^2 - 1)$$

$$a^3 + b^3 = (a + b)(a^2 + ab + b^3)$$

$$= (x^2 + 1)(x + 1)(x - 1)$$

$$10.a. \quad x^6 - 16x^4 = 0 \quad x^4 = 0 \quad \text{or} \quad x + 4 = 0 \quad \text{or} \quad x - 4 = 0$$

$$x^4 (x^2 - 16) = 0 \quad x = 0 \quad x = -4 \quad x = 4$$

$$x^4 (x + 4)(x - 4) = 0 \quad 0, \pm 4$$

$$10.b. \quad (x - 2)(2x + 5)(2x - 5) = 0$$

$$x - 2 = 0 \quad \text{or} \quad 2x + 5 = 0 \quad \text{or} \quad 2x - 5 = 0 \quad 2, \pm \frac{5}{2}$$

$$x = 2 \quad x = -\frac{5}{2} \quad x = \frac{5}{2}$$

$$10.c. \quad (2x + 3)(4x^2 - 6x + 9) = 0$$

$$2x + 3 = 0 \quad \text{or} \quad 4x^2 - 6x + 9 = 0$$

$$x = -\frac{3}{2} \quad \sqrt{b^2 - 4ac} = \sqrt{(-6)^2 - 4(4)9} = \sqrt{36 - 72}$$

the square root of a negative is not real

$$11.a. \quad 3 \sin^2 x = \cos^2 x$$

$$\frac{\sin^2 x}{\cos^2 x} = \frac{1}{3}$$

$$\tan^2 x = \frac{1}{3} \quad \Rightarrow \quad \tan x = \pm \frac{1}{\sqrt{3}} = \pm \frac{\sqrt{3}}{3} \quad \frac{\pi}{6}, \frac{5\pi}{6}, \frac{7\pi}{6}, \frac{11\pi}{6}$$

$$11. b. \quad (1 - \sin^2 x) - \sin^2 x = \sin x$$

$$1 - 2\sin^2 x - \sin x = 0$$

$$0 = 2\sin^2 x + \sin x - 1$$

$$0 = (2\sin x - 1)(\sin x + 1)$$

$$2\sin x - 1 = 0 \quad \text{or} \quad \sin x + 1 = 0 \quad (-\pi < x < \pi)$$

$$\sin x = \frac{1}{2} \qquad \sin x = -1$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6} \qquad x \left(= \frac{3\pi}{2} \right) = -\frac{\pi}{2}$$

$$11. c. \quad \tan x + \sec x = 2 \cos x$$

$$\cos x \left[\frac{\sin x}{\cos x} + \frac{1}{\cos x} = 2 \cos x \right]$$

$$\sin x + 1 = 2 \cos^2 x$$

$$\sin x + 1 = 2(1 - \sin^2 x)$$

$$\sin x + 1 = 2 - 2\sin^2 x$$

$$2\sin^2 x + \sin x - 1 = 0$$

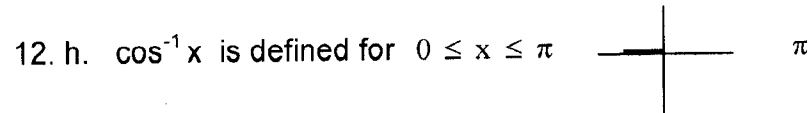
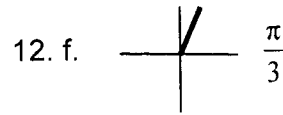
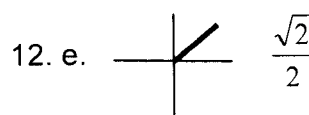
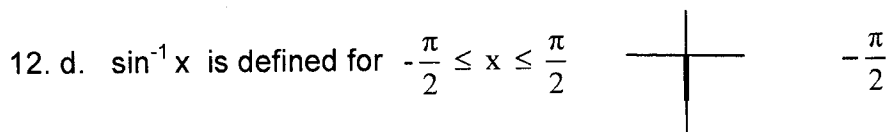
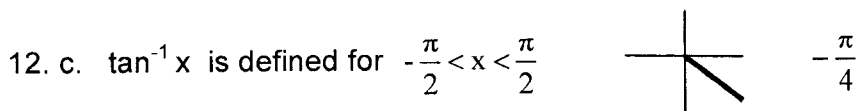
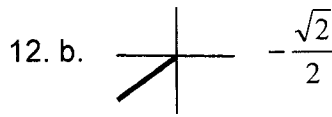
$$(2\sin x - 1)(\sin x + 1) = 0$$

$$2\sin x - 1 = 0 \quad \text{or} \quad \sin x + 1 = 0$$

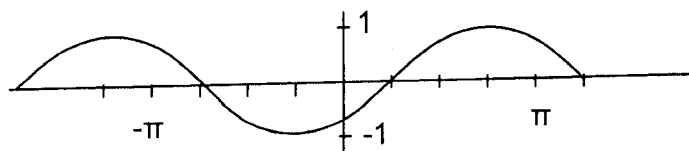
$$\sin x = \frac{1}{2} \qquad \sin x = -1$$

$$x = \frac{\pi}{6}, \frac{5\pi}{6} \qquad x = \frac{3\pi}{2} \quad (\tan x \text{ and } \sec x \text{ are undefined here})$$

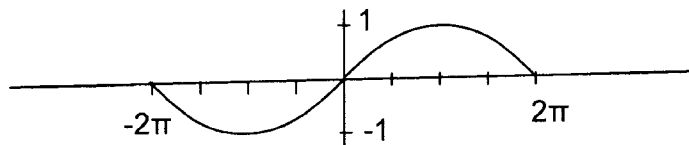
$$x = \frac{\pi}{6} + 2k\pi \text{ or } \frac{5\pi}{6} + 2k\pi \text{ where } k \text{ is any integer}$$



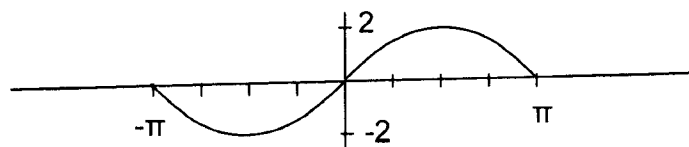
13. a. $\sin\left(x - \frac{\pi}{4}\right)$ $b=1$ $c=\frac{\pi}{4}$ phase shift $= \frac{c}{b} = \frac{\frac{\pi}{4}}{1} = \frac{\pi}{4}$ to right



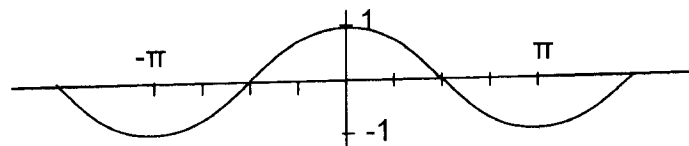
13. b. $\sin \frac{x}{2}$ horizontal stretch 2 times



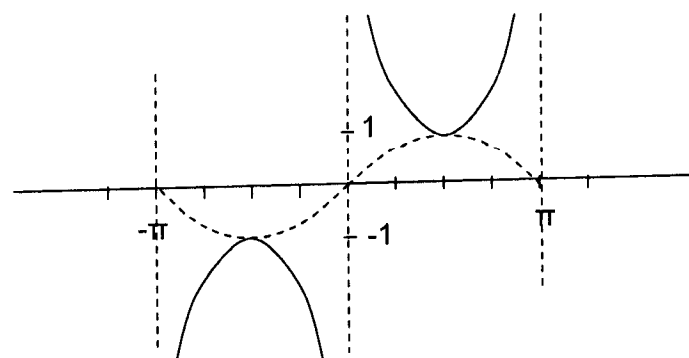
13. c. $2\sin x$ magnitude increased 2 times



13. d. $\cos x$



13. e. $\frac{1}{\sin x} = \csc x$ Draw $\sin x$ curve to help draw $\csc x$ curve, $-\pi$, 0 , and π are asymptotes



14. a. $4x^2 + 12x + 3 = 0$ not factorable - use quadratic formula

$a = 4$ $b = 12$ $c = 3$

$$x = \frac{-12 \pm \sqrt{12^2 - 4(4)3}}{2(4)} = \frac{-12 \pm \sqrt{144 - 48}}{8} = \frac{-12 \pm \sqrt{96}}{8} = \frac{-12 \pm 4\sqrt{6}}{8} = \frac{-3 \pm \sqrt{6}}{2}$$

$$14. b. \quad (2x + 1)(x + 2) = 5$$

$$2x^2 + 4x + x + 2 = 5$$

$$2x^2 + 5x + 2 - 5 = 0$$

$$2x^2 + 5x - 3 = 0$$

$$(2x - 1)(x + 3) = 0$$

$$2x - 1 = 0 \quad \text{or} \quad x + 3 = 0 \quad -3, \frac{1}{2}, (-2 \text{ is not in the domain})$$

$$x = \frac{1}{2} \quad x = -3$$

$$14. c. \quad x(x+1) \left[\frac{x+1}{x} - \frac{x}{x+1} \right] = 0$$

$$(x+1)^2 - x^2 = 0$$

$$x^2 + 2x + 1 - x^2 = 0$$

$$2x + 1 = 0$$

$$x = -\frac{1}{2} \quad (x = 0 \text{ and } -1 \text{ are not in the domain})$$

$$15. a. \quad \begin{array}{r|rrrrrr} -2 & 1 & -4 & 1 & 0 & -7 & 1 \\ & & -2 & 12 & -26 & 52 & -90 \\ \hline & 1 & -6 & 13 & -26 & 45 & -89 \end{array}$$

Remainder -89

$$15. b. \quad \begin{array}{r} x^2 - x + 1 \\ x^3 + 1 \overline{) x^5 - x^4 + x^3 + 2x^2 - x + 4} \\ \underline{x^5} \\ -x^4 + x^3 + x^2 - x + 4 \\ \underline{-x^4} -x \\ x^3 + x^2 + 4 \\ \underline{x^3} + 1 \\ x^2 + 3 \end{array}$$

Remainder $x^2 + 3$

$$16. a. \quad \begin{array}{r|rrrr} 2 & 12 & -23 & -3 & 2 \\ & & 24 & 2 & -2 \\ \hline & 12 & 1 & -1 & 0 \end{array}$$

$$(12x^2 + x - 1) = (4x - 1)(3x + 1)$$

$$12x^3 - 23x^2 - 3x + 2 = (x - 2)(4x - 1)(3x + 1) \quad \left\{ 2, \frac{1}{4}, -\frac{1}{3} \right\}$$

16. b. Rational zero (root) theorem $p = \pm 1$ $q = \pm 12, \pm 6, \pm 4, \pm 3, \pm 2, \pm 1$

$$\frac{p}{q} = \pm \frac{1}{12}, \pm \frac{1}{6}, \pm \frac{1}{4}, \pm \frac{1}{3}, \pm \frac{1}{2}, \pm 1$$

$$\begin{array}{r} \cdot \frac{1}{2} \mid \quad 12 \quad 8 \quad -1 \quad -1 \\ \quad \quad \quad 6 \quad 7 \\ \hline \quad \quad 12 \quad 14 \quad 6 \end{array} \quad \text{Not a solution}$$

$$\begin{array}{r} \cdot \frac{-1}{2} \mid \quad 12 \quad 8 \quad -1 \quad -1 \\ \quad \quad \quad -6 \quad -1 \quad 1 \\ \hline \quad \quad 12 \quad 2 \quad -2 \quad 0 \end{array}$$

$$(12x^2 + 2x - 2) = 2(6x^2 + 1x - 1) = 2(3x - 1)(2x + 1)$$

$$\left\{ -\frac{1}{2}, \frac{1}{3}, -\frac{1}{2} \right\}$$

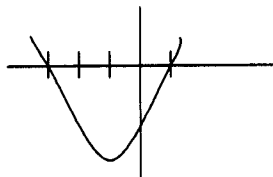
17. a. $x^2 + 2x - 3 \leq 0$

$$(x - 1)(x + 3) \leq 0$$

$$x - 1 = 0 \quad \text{or} \quad x + 3 = 0$$

$$x = 1 \quad \quad x = -3$$

zeros -3, 1



$$[-3, 1] \text{ or } -3 \leq x \leq 1$$

17. b. If $3x - 2 > 0$ $x > \frac{2}{3}$

$$2x - 1 \leq 3x - 2$$

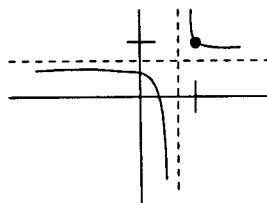
$$1 \leq x$$

If $3x - 2 < 0$

$$x < \frac{2}{3}$$

$$2x - 1 \geq 3x - 2$$

$$1 \geq x$$

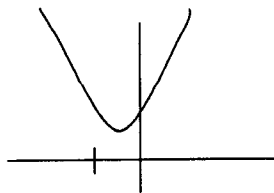


$$x \geq 1 \text{ or } x < \frac{2}{3}$$

17. c. $(x^2 + x + \quad) + 1 > 0$

$$\left(x^2 + x + \frac{1}{4} \right) + 1 - \frac{1}{4} > 0$$

$$\left(x^2 + x + \frac{1}{4} \right) + \frac{3}{4} > 0$$



never less than zero, so all x

18. a. $-1 \leq -x + 4 \leq 1$

$$\begin{array}{r} -4 \quad -4 \quad -4 \\ \hline -5 \leq -x \leq -3 \end{array}$$

$$5 \geq x \geq 3$$

18. b. $5x - 2 = 8$

$$\begin{array}{r} +2 \quad +2 \\ \hline 5x = 10 \end{array}$$

$$x = 2$$

or $-(5x - 2) = 8$

$$-5x + 2 = 8$$

$$\begin{array}{r} -2 \quad -2 \\ \hline -5x = 6 \end{array}$$

$$-5x = 6$$

$$x = -\frac{6}{5}$$

$$\begin{array}{lcl}
 18.c. & 2x+1 = x+3 & \text{or} \quad -(2x+1) = x+3 \\
 & x = 2 & -2x - 1 = x+3 \\
 & & -4 = 3x \\
 & & x = -\frac{4}{3}
 \end{array}$$

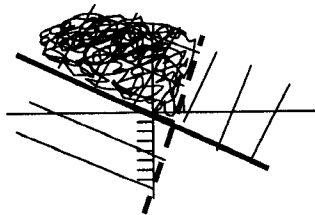
$$\begin{array}{lcl}
 19.a. & m = \frac{-4-3}{2-1} = \frac{-7}{3} & y+4 = \frac{-7}{3}(x-2) \\
 & & 3(y+4) = -7(x-2) \\
 & & 3y+12 = -7x+14 \quad \text{or} \quad 3y = -7x+2 \\
 & & 7x+3y = 2 \quad y = -\frac{7}{3}x + \frac{2}{3}
 \end{array}$$

$$\begin{array}{lcl}
 19.b. & 2x - 3y + 5 = 0 & \perp m = \frac{-3}{2} \quad y-2 = \frac{-3}{2}(x+1) \\
 & -3y = -2x - 5 \quad 0 & 2(y-2) = -3(x+1) \\
 & y = \frac{2}{3}x + \frac{5}{3} \Rightarrow m = \frac{2}{3} & 2y-4 = -3x-3 \\
 & \text{or } m = -\frac{A}{B} = -\frac{2}{-3} = \frac{2}{3} & 3x + 2y = 1
 \end{array}$$

$$19.c. \left(\frac{-1+3}{2}, \frac{4+2}{2} \right) = (1, 3) \quad y=3$$

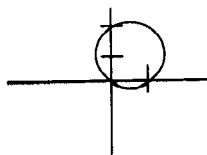
$$\begin{array}{lcl}
 20.a. & y = 3x - 7 & \text{Substitute } x + 5(3x-7) + 3 = 0 \\
 & & x + 15x - 35 + 3 = 0 \\
 & & 16x - 32 = 0 \\
 & & x = 2 \quad y = 3(2) - 7 = -1 \quad (2, -1)
 \end{array}$$

$$\begin{array}{lcl}
 20.b. & y \geq 3x - 7 & \text{and } 5y \geq -x - 3 \\
 & \text{---} & y \geq -\frac{x}{5} - \frac{3}{5}
 \end{array}$$



$$\begin{array}{lcl}
 21.a. & (x-h)^2 + (y-k)^2 = R^2 & \text{where } (h, k) \text{ is the center of the circle and } R \text{ is the radius} \\
 & (x-1)^2 + (y-2)^2 = R^2 & \\
 & (-2-1)^2 + (-1-2)^2 = R^2 & \\
 & 9 + 9 = R^2 & \\
 & 18 = R^2 \Rightarrow (x-1)^2 + (y-2)^2 = 18 &
 \end{array}$$

21. b. Uses a Geometry rule that says the segment perpendicular to the midpoint of a chord must be a diameter of the circle (i. e. go through the center of the circle)



The midpoint of (0, 0) & (0, 2) is (0, 1)
and the midpoint of (0, 0) & (1, 0) is (1/2, 0)
so the center of the circle is (1/2, 1)

continued next page

21.b. cont. $(x-h)^2 + (y-k)^2 = R^2$ where (h, k) is the center of the circle and R is the radius

$$\left(x - \frac{1}{2}\right)^2 + (y-1)^2 = R^2 \quad \text{using } (0,0) \text{ as the point on the circle}$$

$$\frac{1}{4} + 1 = R^2$$

$$\frac{5}{4} = R^2 \Rightarrow \left(x - \frac{1}{2}\right)^2 + (y-1)^2 = \frac{5}{4}$$

22.a. $(x-h)^2 + (y-k)^2 = R^2$ where (h, k) is the center of the circle and R is the radius

$$(x^2 + 6x) + (y^2 - 4y) + 3 = 0 \quad \text{Complete the squares}$$

$$(x^2 + 6x + 9) + (y^2 - 4y + 4) + 3 - 9 - 4 = 0$$

$$(x+3)^2 + (y-2)^2 - 10 = 0$$

$$(x+3)^2 + (y-2)^2 = 10 \Rightarrow \text{center } (-3, 2) \text{ \& Radius } \sqrt{10}$$

b. The slope of the radius from $(-3, 2)$ to $(-2, 5)$ is

$$m = \frac{5-2}{-2-(-3)} = \frac{3}{1} = 3 \quad \text{to be tangent to the circle a line must be perpendicular to the radius}$$

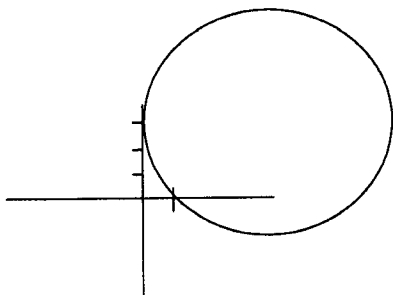
so the slope of the tangent line is $-\frac{1}{3}$

$$(y-5) = -\frac{1}{3}(x+2)$$

$$3y - 15 = -x - 2$$

$$x + 3y = 13$$

23.



The center is on $y=3$ or $k=3$

two points on the circle is $(1, 0)$ & $(0, 3)$

$$(1-h)^2 + (0-3)^2 = R^2$$

$$(0-h)^2 + (3-3)^2 = R^2$$

$$(1-h)^2 + (0-3)^2 = (0-h)^2 + (3-3)^2$$

$$1 - 2h + h^2 + 9 = h^2$$

$$10 - 2h = 0$$

$$10 = 2h$$

$$h = 5$$

b. Center is $(5, 3)$

$$(0-5)^2 + (3-3)^2 = R^2$$

$$25 = R^2$$

$$(x-5)^2 + (y-3)^2 = 25$$

a. Call the other x -intercept $(a, 0)$

$$(a-5)^2 + (0-3)^2 = 25$$

$$a^2 - 10a + 25 + 9 = 25$$

$$a^2 - 10a + 9 = 0$$

$$(a-9)(a-1) = 0 \Rightarrow a=1 \text{ or } 9 \quad (9, 0)$$

24. Distance from A = $\sqrt{(x+1)^2 + (y-1)^2}$ and Distance from B = $\sqrt{(x-2)^2 + (y+1)^2}$

Distance from A = 3(Distance from B)

$$\sqrt{(x+1)^2 + (y-1)^2} = 3\sqrt{(x-2)^2 + (y+1)^2}$$

$$(x+1)^2 + (y-1)^2 = 9((x-2)^2 + (y+1)^2)$$

$$[x^2 + 2x + 1 + y^2 - 2y + 1] = 9[x^2 - 4x + 4 + y^2 + 2y + 1]$$

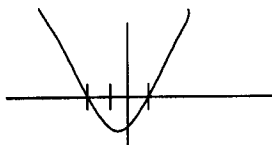
$$x^2 + 2x + 1 + y^2 - 2y + 1 = 9x^2 - 36x + 36 + 9y^2 + 18y + 9$$

$$0 = 8x^2 + 8y^2 - 38x + 20y + 43$$

25. a. $x^2 + x - 2 > 0$

$$(x+2)(x-1) > 0$$

Domain $(-\infty, -2)$ and $(1, \infty)$



25. b. i. Domain all real numbers, Range 7

25. b. ii. Domain all real numbers except $x = \frac{1}{2}$ (vertical asymptote),

Range all real numbers except $\frac{5}{2}$ (horizontal asymptote)

26. $x > 0 \quad f(x) = \frac{x}{x} = 1 \quad x < 0 \quad f(x) = \frac{x}{-x} = -1 \quad x = 0 \quad f(x) \text{ is undefined}$

Domain all real numbers except 0, Range $\{-1, 1\}$

27. a. $\frac{[2(x+h)+3] - [2x+3]}{h} = \frac{2x+2h+3 - 2x-3}{h} = \frac{2h}{h} = 2$

27. b. $\frac{\left[\frac{1}{x+h+1}\right] - \left[\frac{1}{x+1}\right]}{h} = \frac{(x+h+1)(x+1)}{(x+h+1)(x+1)} \left[\frac{\left[\frac{1}{x+h+1}\right] - \left[\frac{1}{x+1}\right]}{h} \right] = \frac{(x+1) - (x+h+1)}{h(x+h+1)(x+1)}$

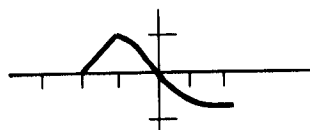
$$= \frac{x+1-x-h-1}{h(x+h+1)(x+1)} = \frac{-h}{h(x+h+1)(x+1)} = \frac{-1}{(x+h+1)(x+1)}$$

27. c. $\frac{(x+h)^2 - x^2}{h} = \frac{x^2 + 2xh + h^2 - x^2}{h} = \frac{2xh + h^2}{h} = 2x + h$

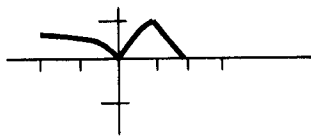
28. a. Shifts one unit to left



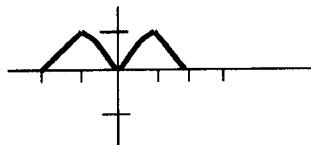
28. b. Reflects across y-axis



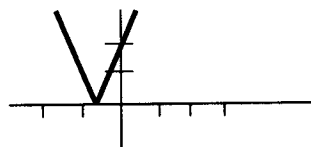
28. c. all positive y values



28. d. negative x-axis is reflection of positive x-axis



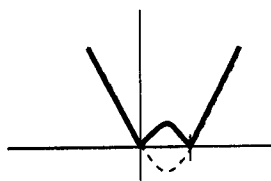
29. a. Draw $y = 3x + 2$ – do not go below the x-axis, reflect across $x = -\frac{2}{3}$



29. b. $(x^2 + x + \quad)$

$$\left(x^2 + x + \frac{1}{4}\right) + -\frac{1}{4}$$

$$\left(x + \frac{1}{2}\right)^2 - \frac{1}{4}$$



30. a.

Less than 4 indicates the parabola faces down

$$\begin{aligned} y &= -(x+1)(x-3) = -(x^2 - 2x - 3) = -(x^2 - 2x - \quad) + 3 \\ &= -(x^2 - 2x + 1) + 3 + 1 \\ &= -(x^2 - 1)^2 + 4 \end{aligned}$$

30. b.

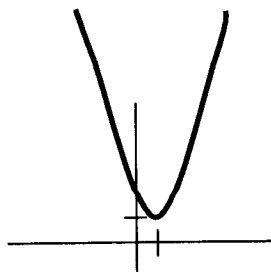
$$y = 2x^2 - 4x + 3$$

$$y = 2(x^2 - 2x \quad) + 3$$

$$y = 2(x^2 - 2x + 1) + 3 - 2$$

$$y = 2(x-1)^2 + 1$$

vertex (1,1) goes up $2x$ faster than x^2



31. a.

$$x = t + 1 \quad t = x - 1$$

$$y = (x-1)^2 - (x-1)$$

$$y = x^2 - 2x + 1 - x + 1$$

$$y = x^2 - 3x + 2$$

31. b.

$$x = \sqrt[3]{t} - 1 \quad \sqrt[3]{t} = x + 1$$

$$t = (x+1)^3$$

$$y = (x+1)^3 [(x+1)^3 - 1] \quad \text{Acceptable Answer}$$

$$y = (x+1)^3 [x^3 + 3x^2 + 3x + 1 - 1]$$

$$y = (x+1)^3 [x^3 + 3x^2 + 3x]$$

$$y = x(x+1)^3 [x^2 + 3x + 3]$$

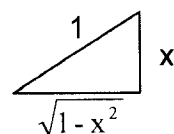
31. c. $x = \sin t$ $t = \sin^{-1} x$

$$y = \cos(\sin^{-1} x)$$

$$y = \sqrt{1 - x^2}$$

$$y^2 = 1 - x^2$$

$$x^2 + y^2 = 1$$



32. a. $x = 2y + 3$

$$2y = x - 3$$

$$y = \frac{x-3}{2}$$

32. b. $x = \frac{y+2}{5y-1}$

$$(5y-1)x = y+2$$

$$5yx - x = y + 2$$

$$5yx - y = x + 2$$

$$y(5x-1) = x+2$$

$$y = \frac{x+2}{5x-1}$$

32. c. $x = y^2 + 2y - 1$

$$0 = y^2 + 2y - 1 - x$$

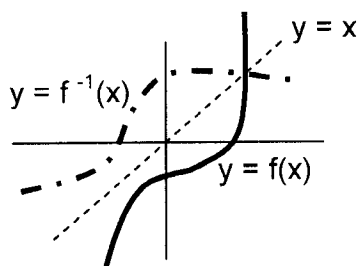
$$0 = y^2 + 2y - (1+x)$$

$$a=1, b=2, c=-(1+x)$$

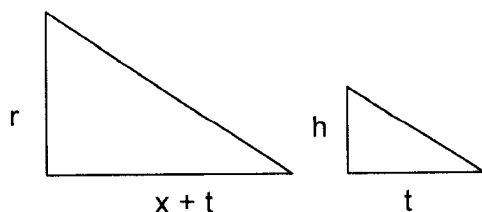
$$y = \frac{-2 \pm \sqrt{4 - 4(-(1+x))}}{2} = \frac{-2 \pm \sqrt{4 + 4 + 4x}}{2} = \frac{-2 \pm \sqrt{8 + 4x}}{2} = \frac{-2 \pm \sqrt{4(2+x)}}{2} = -1 \pm \sqrt{2+x}$$

When $x=0$ in the original equation $y=-1$, so x is limited to $x > -1$ for the inverse.

33. reflects across $y = x$



34. a.



Similar Triangles

$$\frac{x+t}{t} = \frac{r}{h}$$

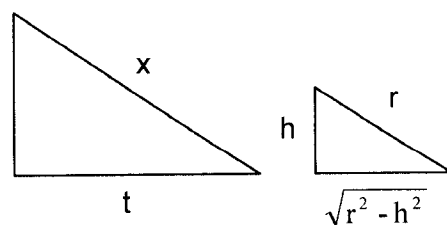
$$x+t = \frac{rt}{h}$$

$$x = \frac{rt}{h} - t$$

$$x = \frac{rt}{h} - \frac{th}{h}$$

$$x = \frac{t(r-h)}{h}$$

b.



Similar Triangles

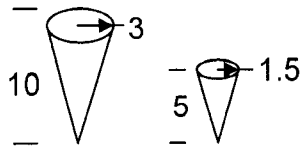
$$\frac{x}{r} = \frac{t}{\sqrt{r^2 - h^2}}$$

$$x = \frac{rt}{\sqrt{r^2 - h^2}}$$

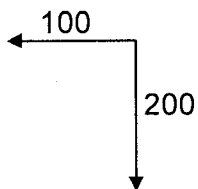
35. a. Area of the square $= (2r)^2 = 4r^2$ Area of the square $= \pi r^2$

Area inside the square but outside the circle $= 4r^2 - \pi r^2$ ratio $= \frac{4r^2 - \pi r^2}{4r^2} = \frac{4 - \pi}{4} = 1 - \frac{\pi}{4}$

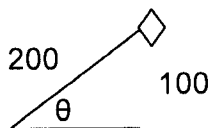
35. b. perimeter $= r + 2r + r + \frac{2\pi r}{2} = 4r + \pi r$



35. c. Area $= \pi (1.5)^2 = 2.25 \pi$



35. d. Distance $= \sqrt{100^2 + 200^2} = \sqrt{100^2 + 2^2 (100^2)} + \sqrt{(1+4)(100^2)} = 100\sqrt{5}$ km



35. e. $\sin \theta = \frac{100}{200} \Rightarrow \theta = \sin^{-1} \left(\frac{100}{200} \right) = \frac{\pi}{6}$ or 30°